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Hydraulic fluid power - Determination of derived displacement of positive displacement pumps and motors - Part 1: One-step and two-step Toet-method

WD stage

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A model manuscript of a draft International Standard (known as "The Rice Model") is available at https://www.iso.org/iso/model_document-rice_model.pdf

Contents

Foreword

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This document was prepared by Technical Committee ISO/TC 131, *Fluid power systems*, Subcommittee SC 08, *Product testing*.

A list of all parts in the ISO 8426 series can be found on the ISO website.

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Introduction

In hydraulic fluid power systems, power is transmitted and controlled through a liquid under pressure within an enclosed circuit. Two types of components of such systems are the positive displacement pumps and motors. One of the technical parameters of these components is the derived displacement, also known as derived displacement. This document is intended to describe the test procedure and analytical approach of the Toet-method for determining the derived displacement of hydraulic fluid power positive displacement pumps and motors.

Hydraulic fluid power - Determination of derived displacement of positive displacement pumps and motors - Part 1: One-step and two-step Toet-method

1 Scope

This document specifies the Toet methods for the determination of the derived displacement of hydraulic fluid power positive displacement pumps and motors under steady-state conditions. A single value for the derived displacement is determined from measurements at multiple shaft speeds.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 4409:2019, *Hydraulic fluid power — Positive-displacement pumps, motors and integral transmissions — Methods of testing and presenting basic steady state performance*

ISO 5598:2020, *Fluid power systems and components — Vocabulary*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 5598:2020, the following apply:

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at http://www.electropedia.org/

3.1
unit
positive displacement pump or motor

3.2
ordinary least squares regression
common technique for estimating coefficients of linear regression equations which describe the relationship between one or more independent quantitative variables and a dependent variable.

Note 1 to entry: "Least squares" corresponds to the minimum square error.

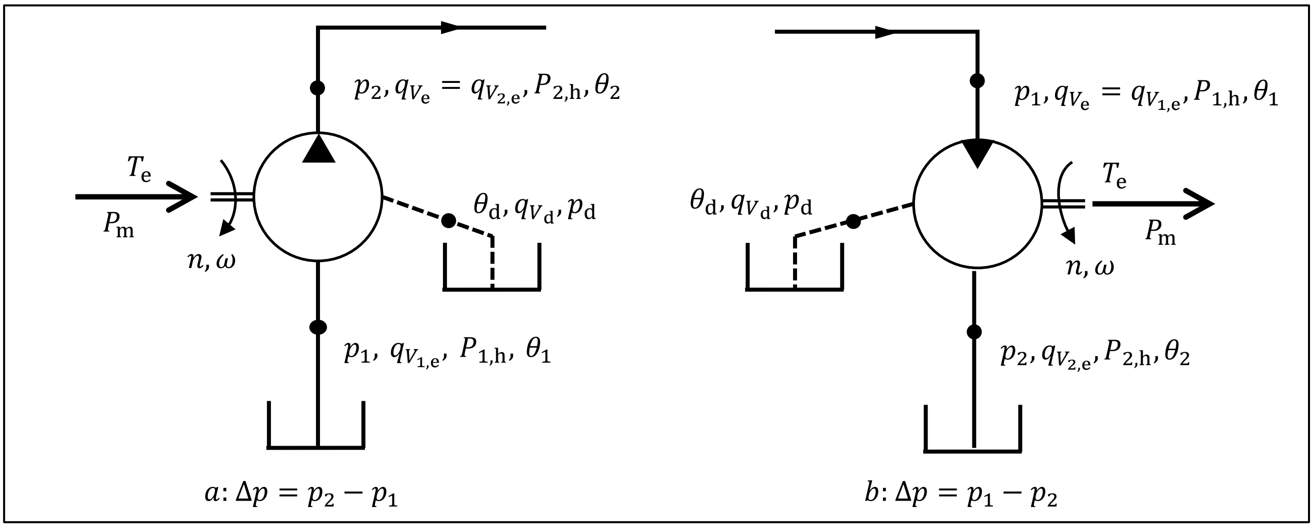
4 Symbols, subscripts, and graphical symbols

The general symbols used throughout this standard are shown in Table 1 based on ISO 4391:1983 (1). Graphical symbols are depicted in accordance with ISO 1219 series (2).

An example diagram showing the use of symbols, subscripts, and graphical symbols is shown in Figure 1. Figure 1 depicts the measured location of each symbol with subscripts and the use of graphical symbols for pumps and motors.

Table 1—Terms, symbols, description, and units.

Term	Symbol	Description	Dimensions	Practical Units
case pressure	p_{d}	The pressure in the drain port of the pump or motor	$mL^{-1}t^{-2}$	bar
differential pressure	Δp	See ISO 5598:2020, 3.2.211	$mL^{-1}t^{-2}$	bar
derived displacement	V_{i}	See ISO 5598:2020, 3.2.200	m^3	cm^3/rev
displacement	V	See ISO 5598:2020, 3.219	m^3	cm^3/rev
high pressure flow	$q_{V_{\mathrm{e}}}$	Flow rate at the high pressure port	$m^3 \cdot T^{-1}$	L/min
high pressure flow of a pump	$q_{V_{2,\mathrm{e}}}$	Flow rate at the high pressure port of a pump	$m^3 \cdot T^{-1}$	L/min
high pressure flow of a motor	$q_{V_{1,\mathrm{e}}}$	Flow rate at the high pressure port of a motor	$m^3 \cdot T^{-1}$	L/min
inlet pressure	p_1	See ISO 5598:2020, 3.2.397	$mL^{-1}t^{-2}$	bar
inlet temperature	θ_1	Temperature at the inlet of the pump or motor	°C	°C
outlet pressure	p_2	See ISO 5598:2020, 3.2.510	$mL^{-1}t^{-2}$	bar
shaft speed	n	See ISO 4391:1983, 7.1.6, 11.2, and 11.4	T^{-1}	rev/min



Key

a Pump differential pressure

b Motor differential pressure

Figure 1—Example use of symbols, subscripts, and graphical symbols for pumps and motors.

5 Test Procedure

5.1 Requirements

5.1.1

Measurement accuracy class according to ISO 4409:2019 clause A.2 shall be used for all measured quantities.

5.1.2

ISO 4409:2019 clause 5.1 and its subclauses shall be used for test procedure requirements.

5.2 Positive displacement pump test procedure

ISO 4409:2019 clause 5.2 and its subclauses shall be used for the pump testing procedure.

5.3 Positive displacement motor test procedure

ISO 4409:2019 clause 5.3 and its subclauses shall be used for the motor testing procedure.

5.4 Number of steady-state shaft speeds and differential pressures

5 or more shaft speeds and 5 or more differential pressures should be used. This means a minimum of 25 test points should be used to calculate derived displacement. The shaft speed may be increased in equal increments from minimum shaft speed to maximum shaft speed. The differential pressure may be increased in equal increments from minimum differential pressure to maximum differential pressure.

5.5 Variable displacement units

The data for performing the estimation of the derived displacement of variable displacement pumps and motors shall be obtained following ISO 4409:2019 clause 5.2.4. Derived displacement of a unit at partial displacement shall be estimated using one or both procedures presented in clause 6.

6 Calculation of derived displacement

6.1 General

6.1.1

The mathematical definition of the derived displacement is shown for a pump and motor in Formula 1 and Formula 2 respectively. Expressing Formula 1 and Formula 2 in words, the derived displacement is the slope of the high pressure flow with respect to shaft speed evaluated at zero differential pressure. One of the fundamental implications of the Toet methods is derived displacement is assumed independent of speed and differential pressure according to (3) clause 4 and (4) clause 4. Therefore, the task of finding the derived displacement is to find the slope of the high pressure flow with respect to shaft speed at zero differential pressure.

$$V_i = \left(\frac{\partial q_{V_e}}{\partial n}\right)_{\Delta p=0} = \left(\frac{\partial q_{V_{2,e}}}{\partial n}\right)_{\Delta p=0} \quad (1)$$

Where

$q_{V_\mathrm{e}}, q_{V_{2,\mathrm{e}}}$	is the high pressure outlet flow of a pump
n	is the shaft speed
V_i	is the derived displacement
$(\ )_{\Delta p=0}$	implies evaluating the expression at a Δp of 0

$$V_i = \left(\frac{\partial q_{V_e}}{\partial n}\right)_{\Delta p=0} = \left(\frac{\partial q_{V_{1,e}}}{\partial n}\right)_{\Delta p=0} \quad (2)$$

Where

$q_{V_\mathrm{e}}, q_{V_{1,\mathrm{e}}}$	is the high pressure inlet flow of a motor
n	is the shaft speed
V_i	is the derived displacement
$(\ )_{\Delta p=0}$	implies evaluating the expression at a Δp of 0

6.1.2

The determination of the derived displacement shall be obtained using either of the methods described in clauses 6.2, or 6.3. The Two-step Toet Method in Clauses 6.2 and The One-step Toet Method in 6.3 are derived from the same principles and produce the same derived displacement.

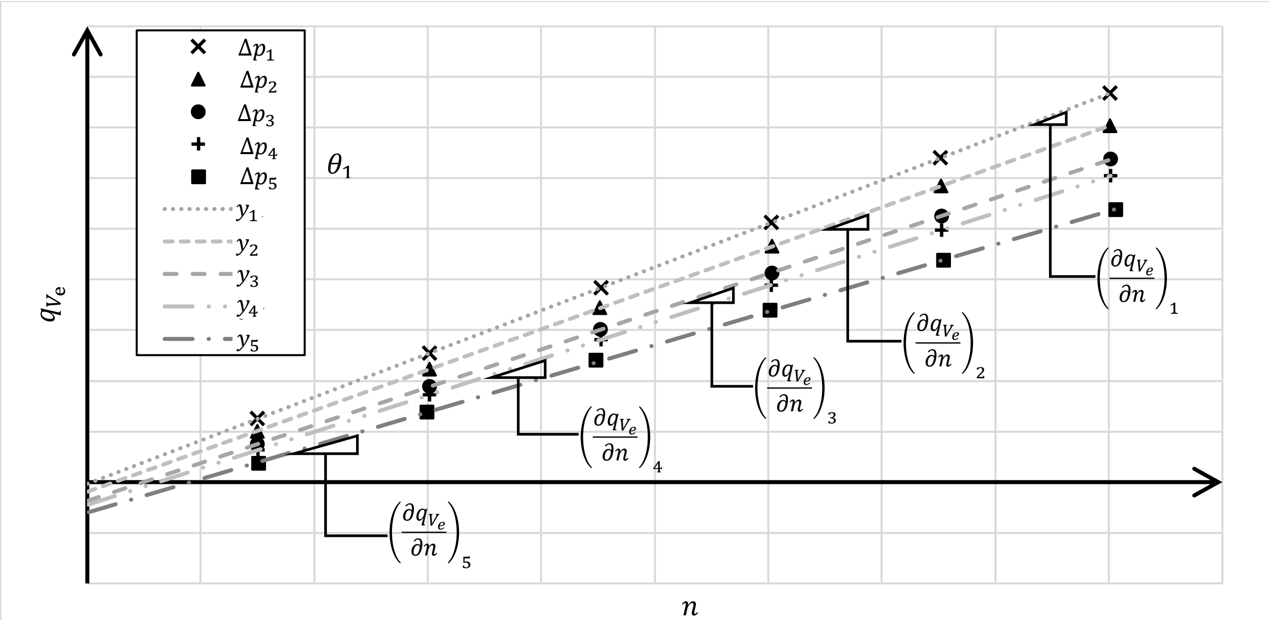
6.2 The Two-step Toet Method

6.2.1

The Two-step Toet Method was first described in (3) and later revised in (4). The steps below shall be used to calculate the derived displacement using the Two-Step Toet Method:

1. The high pressure flow, q_{V_e}, shall be measured at various shaft speeds, n for a nominal differential pressure, Δp_1; An ordinary least squares regression of a line shall be performed between high pressure flow and shaft speed to find the slope $\left(\frac{\partial q_{V_e}}{\partial n}\right)_1$. The regression should be done

automatically using a spreadsheet program or statistical software; the regression may be carried out via the formulas in clause D.1. Repeat this step for all the nominal differential pressures.



Key

n shaft speed

q_{V_e} high pressure flow

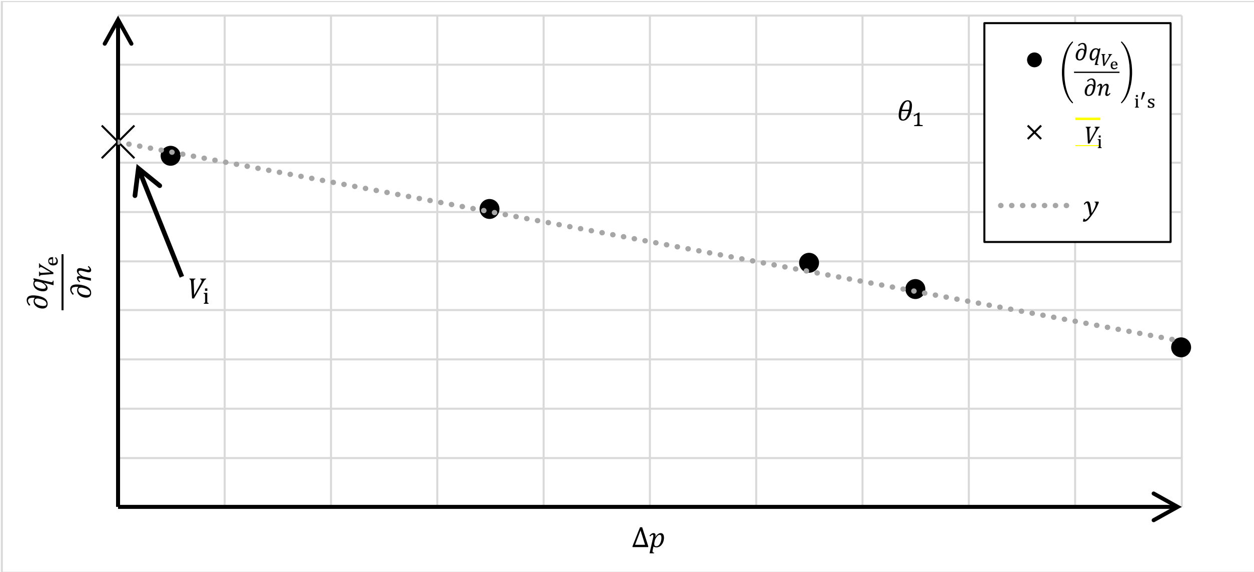
$\left(\frac{\partial q_{V_\mathrm{e}}}{\partial n}\right)_i$ the slope of fitted line corresponding to Δp_i where $1 \leq i \leq 5$. This is displacement at Δp_i.

θ_1 constant inlet temperature

y_i linear regression line corresponding to Δp_i where $1 \leq i \leq m$, m

Figure 2—The first step of the Two-step Toet Method

2. An ordinary least squares regression of a line shall be used to find the intercept of the displacements $\left(\frac{\partial q_{V_\mathrm{e}}}{\partial n}\right)_i$ with respect to the Δp_i's. The intercept is derived displacement, V_i. This regression should be done automatically using a spreadsheet program or statistical software; the regression may be carried out via the formulas in clause D.1. This step is depicted in Figure 3.



Key

Δp	differential pressure
$\frac{\partial q_{V_e}}{\partial n}$	displacement
$\left(\frac{\partial q_{V_e}}{\partial n}\right)_{i's}$	slopes/displacements found in step one
θ_1	constant inlet temperature
V_i	derived displacement
y	linear regression line

Figure 3—The second step of the Two-Step Toet Method

6.2.2

For more information on guidance related to regression, see Annex C Residual and regression metrics.

6.3 The One-step Toet Method

6.3.1

The One-step Toet Method synthesizes the Two-step Toet Method using the underlying mathematical principles applied in one-step; the details of the derivation are in Annex B.

6.3.2

The high pressure flow, q_{V_e}, shall be measured at the chosen nominal shaft speeds, n and differential pressures, Δp. The derived displacement shall be obtained by ordinary least square regression of Formula 3 to find the derived displacement, V_i which is C_3. All the coefficients C_1 to C_4 shall be obtained from the regression. The regression should be carried out by a spreadsheet program or other statistical software.

$$q_{V_e} = \underbrace{C_1 \cdot n \cdot \Delta p + C_2 \cdot \Delta p} + \underbrace{C_3 \cdot n} + \underbrace{C_4} \quad (3)$$

Where:

$C_1 \cdot n \cdot \Delta p + C_2 \cdot \Delta p$ are loss terms

$C_3 \cdot n$ is the theoretical term, $V_i n$ C_4 is the bias term	

6.3.3

For more information on guidance related to regression, see Annex C Residual and regression metrics.

7 Test Report

7.1 General

7.1.1

This clause may be used as a guideline for reporting the results of the test and analysis. For an example pump test report, see A.1. For an example test report, see A.2.

7.1.2

A minimal test report should be written and should include the following information:

a) Test time and location;
b) description of the unit under test, including model, serial number, and date of manufacture if available;
c) shaft speed operating range;
d) differential pressure operating range;
e) displacement range if applicable and value of current fixed displacement (i.e. displacement is locked);
f) measurement accuracy class (see ISO 4409:2019 A.1 and A.2);
g) test circuit filtration details (see ISO 4409:2019 5.1.1);
h) description of the test circuit, including the location of flow-meters (see ISO 4409:2019 Figure 1, Figure 2, and Figure 3);
i) the nominal ambient temperature of the test area (see ISO 4409:2019 5.1.1);
j) Nominal inlet fluid test temperature, θ_1 (see ISO 4409:2019 5.1.4);
k) Nominal test fluid properties. This includes, at a minimum, fluid name and the properties of kinematic viscosity and density at the nominal inlet temperature, θ_1, and atmospheric pressure; (see ISO 4409:2019 5.1.2 and 5.1.4);
l) nominal case pressure, p_d, if appropriate (see ISO 4409:2019 5.1.5);
m) the nominal time interval for flow rate measurements (see ISO 4409:2019 5.1.6);
n) which method(s) was used for calculating derived displacement from clause 6, the Two-step Toet Method, and/or the One-step Toet Method.
o) p_1 inlet pressure;
p) p_2 outlet pressure;
q) n shaft speed;
r) θ_1 test fluid temperature at the inlet.
s) Δp the differential pressure across the test unit;
t) $q_{V_{2,\mathrm{e}}}$ or $q_{V_{1,\mathrm{e}}}$ high pressure flow (see ISO 4409:2019 5.2.1.1, 5.2.1.2, or 5.3.1);
u) V_i calculated derived displacement;
v) the regression equation(s) with terms, values of coefficients, and correlation coefficient(s) for the method(s) used.

Annex A
(informative)
Test report examples

A.1 Pump test report example

A.1.1 Raw Data

Table A.1 shows the raw data for analysis in the pump test report example.

Table A.1—Raw data for pump example

Nominal Delta Pressure [bar]	Nominal Shaft Speed [rev/min]	Inlet Pressure [bar]	Outlet Pressure [bar]	Shaft Speed [rev/min]	Inlet Temperature [°C]	Delta Pressure [bar]	High Pressure Flow [L/min]
10	300	-0.10	10.01	299.4	49.6	10.1	25.64
10	600	-0.06	10.01	602.2	49.7	10.1	51.65
10	900	0.04	9.99	904.9	49.7	10.0	77.76
10	1200	-0.04	9.99	1205.3	50.3	10.0	103.70
10	1500	0.08	10.01	1502.6	49.7	9.9	129.46
10	1800	-0.03	10.00	1801.7	49.6	10.0	155.31
70	300	-0.02	69.87	299.8	50.1	69.9	24.24
70	600	0.03	70.00	602.2	50.0	70.0	50.43
70	900	0.02	69.92	903.1	49.8	69.9	76.11
70	1200	-0.07	70.47	1205.9	50.0	70.5	102.05
70	1500	-0.07	70.41	1503.8	49.6	70.5	127.37
70	1800	-0.05	70.46	1801.8	50.1	70.5	152.70
130	300	-0.06	130.45	299.2	50.0	130.5	22.88
130	600	0.00	130.05	602.8	49.6	130.1	48.43
130	900	-0.03	130.35	904.0	50.3	130.4	73.84
130	1200	0.07	130.24	1206.0	49.8	130.2	99.00
130	1500	-0.04	129.81	1505.0	49.7	129.9	124.41
130	1800	-0.03	129.87	1802.5	49.7	129.9	149.73
150	300	-0.07	149.99	300.6	50.4	150.1	21.63
150	600	-0.09	150.39	602.8	49.6	150.5	46.94
150	900	0.01	149.84	904.7	49.7	149.8	72.01
150	1200	-0.06	150.43	1205.0	49.8	150.5	96.90
150	1500	-0.01	150.38	1505.1	49.7	150.4	121.79
150	1800	-0.03	150.29	1803.1	50.1	150.3	146.68
210	300	0.05	210.97	301.7	49.6	210.9	20.03
210	600	-0.10	210.91	598.6	49.6	211.0	45.02
210	900	0.04	209.09	895.4	49.9	209.1	69.76
210	1200	-0.05	209.28	1202.4	49.7	209.3	94.24
210	1500	0.00	209.81	1507.9	50.4	209.8	118.81
210	1800	-0.08	209.48	1811.6	49.9	209.6	143.45

A.1.2 Example report

Test time: 2020-03-24 13:03
Test location: City, State or Province, Country
Unit Tested: ACME Series XYZ Fixed Displacement Pump, model 365, serial 1234ABCD
Shaft speed range: [300 rev/min, 1800rev/min]
Differential pressure range: [0 bar, 210 bar]
Measurement Accuracy Class: A
Test circuit filtration details: 23 micron
Description of test circuit:

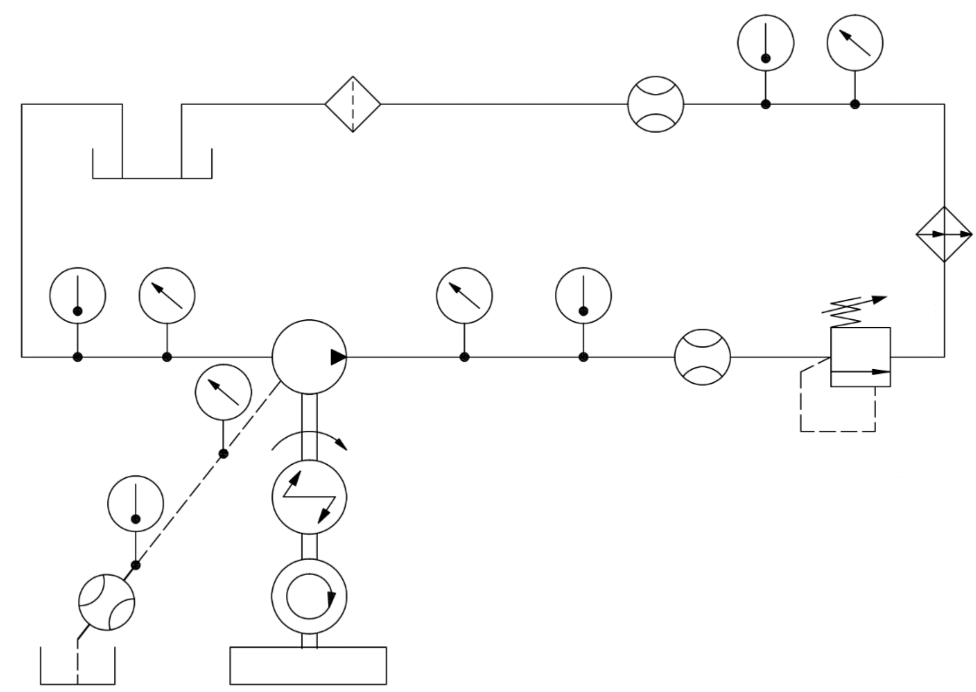
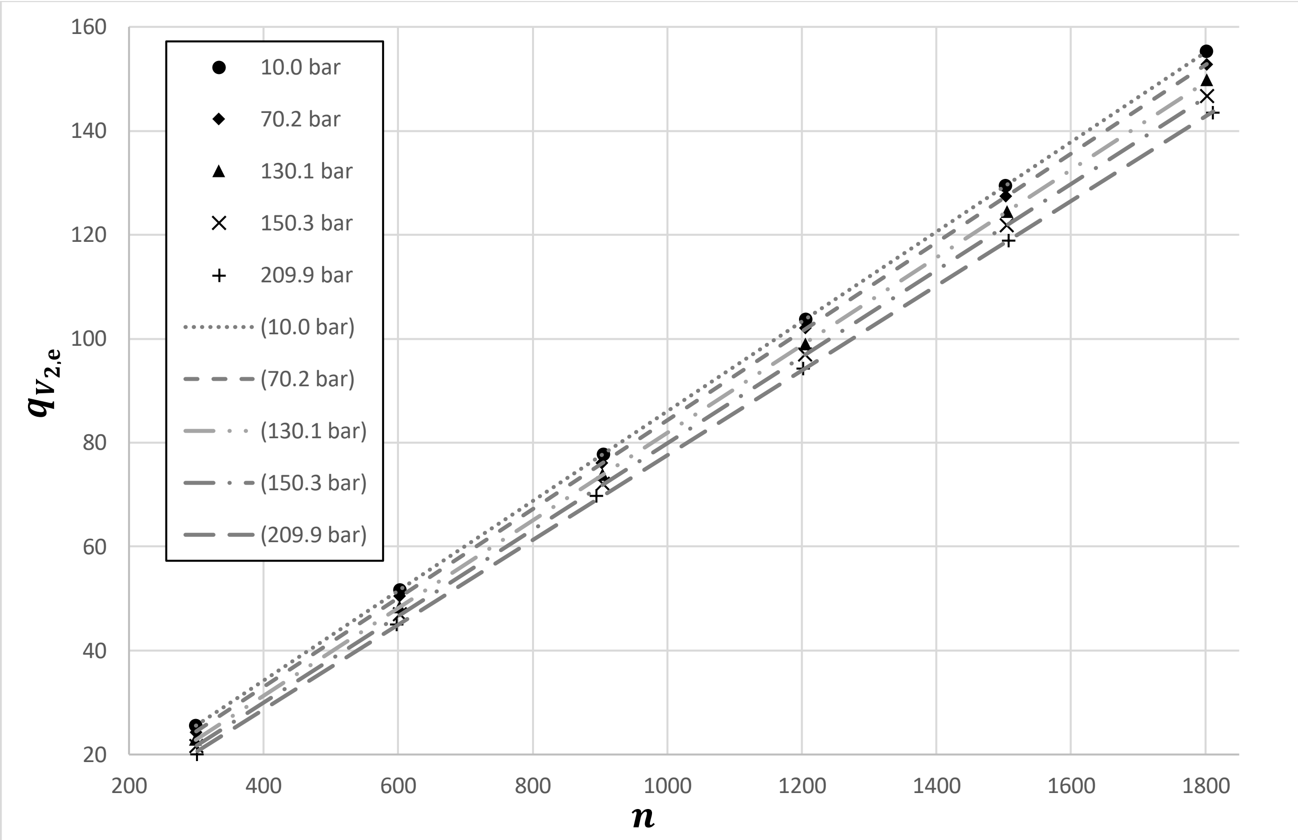


Figure A.1—Example test circuit for an open circuit pump unit

Nominal ambient temperature: 20°C
Nominal inlet temperature: $\theta_1 = 50°C$
Test fluid: ABCDOil-46, kinematic viscosity of $30cSt$ at 50°C, density of $860kg/m^3$ at 50°C
Nominal case pressure: $p_\mathrm{d} = 1.5bar$
Time interval for flow rate measurement: $10sec$
Method for calculating derived displacement:
Both Two-step Toet Method and the One-step Toet Method were used to calculate derive displacement as shown in A.1.2.1 and A.1.2.2 respectively.

A.1.2.1 Two-step Toet Method Results

Figure A.2 and Figure A.3 show the graphs first and second step respectively. Table A.1 and Table A.2 are the results of the first and second step respectively.



Key

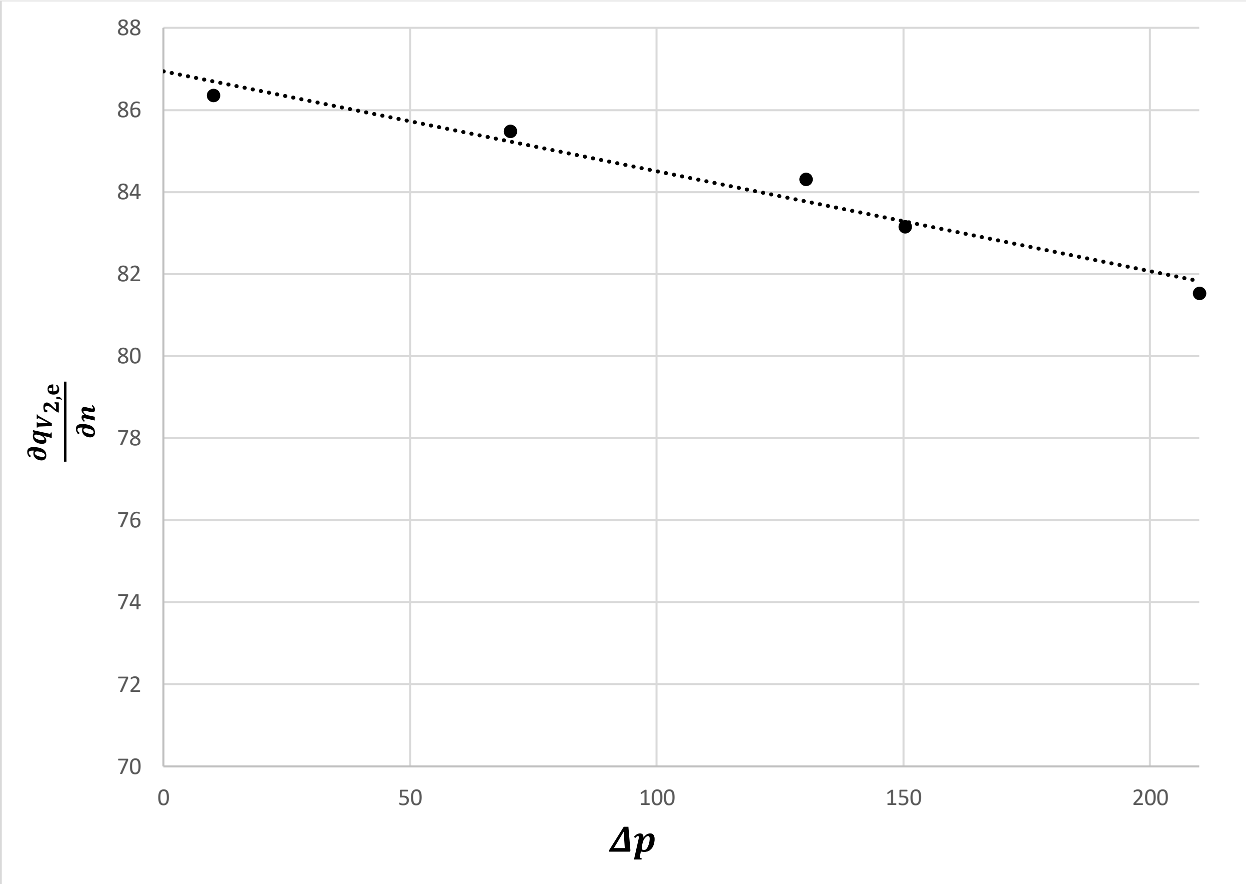
$q_{V_{2,e}}$ high pressure flow at the outlet of the pump, L/min.

n shaft speed, rev/min.

Figure A.2—First step of Two-step Toet Method for the pump example

Table A.1—Results for first step of the pump example

Nominal Delta Pressure [bar]	Delta Pressure Average [bar]	Slope of Fitted Line [cm³/rev]	Intercept of Fitted Line [L/min]	Equation	Correlation Coefficient R^2
10	10.0	86.341	-5.1390	$q_{\mathrm{v}_{\mathrm{e},2}}$ =86.341*n/1000+(-5.1390)	0.9999981
70	70.2	85.482	-5.5299	$q_{\mathrm{v}_{\mathrm{e},2}}$=85.482*$n$/1000+(-5.5299)	0.9999904
130	130.2	84.307	-5.6907	$q_{\mathrm{v}_{\mathrm{e},2}}$=84.307*$n$/1000+(-5.6907)	0.9999902
150	150.3	83.148	-5.2557	$q_{\mathrm{v}_{\mathrm{e},2}}$=83.148*$n$/1000+(-5.2557)	0.9999974
210	209.9	81.525	-3.9552	$q_{\mathrm{v}_{\mathrm{e},2}}$=81.525*$n$/1000+(-3.9552)	0.9998639



Key

$\frac{\partial q_{V_{2,e}}}{\partial n}$ displacement

Δp differential pressure

Figure A.3—Second step of Two-step Toet Method for pump example

Table A.2—Results of second step for pump example

Slope [(cm³/rev)/bar]	Intercept, $\boldsymbol{V}_{\mathbf{i}}$ [cm³/rev]	Equation	Correlation Coefficient $\boldsymbol{R}^{\mathbf{2}}$
-0.02433	86.937	V=-0.02433*Δp +86.937	0.959491301

A.1.2.2 One-step Toet Method Results

The results for the one step method are shown in Table A.3.

Table A.3—Results for the One-step Toet Method for the pump example

C_1 [(cm³/rev)/(rev/min*bar)]	C_2 [L/bar]	C_3, V_i [cm³/rev]	C_4 [L/min]	Equation	Correlation Coefficient R^2
-0.0245	-0.01910	86.952	-0.0402	$q_{v_{e,2}}$= -0.0245*n/1000*Δp +(-0.01910)*Δp +86.952*n/1000 +(-0.0402)	0.9998347

A.2 Motor test report example

A.2.1 Raw data

Table A.4 shows the raw data for analysis in the motor test report example.

Table A.4—Raw Data for motor example.

Nominal Differential pressure [bar]	Nominal Shaft Speed [rpm]	Outlet Pressure [bar]	Inlet Pressure [bar]	Shaft Speed [rpm]	Inlet Temperature [°C]	Differential pressure [bar]	High Pressure Flow [L/min]
30	300	1.98	31.5	300.8	50.5	29.5	31.68
30	600	1.98	31.7	600.8	50.3	29.7	59.32
30	900	1.97	32.1	901.7	50.3	30.1	86.70
30	1200	1.98	32.2	1200.9	50.3	30.2	113.94
30	1500	1.99	31.6	1498.4	50.3	29.6	141.44
70	300	1.97	71.9	301.7	50.2	69.9	29.66
70	600	1.98	71.9	601.7	50.4	69.9	56.77
70	900	1.97	71.8	902.6	50.2	69.8	83.60
70	1200	1.97	71.6	1201.8	50.3	69.6	110.44
70	1500	1.98	71.7	1498.5	50.4	69.7	137.00
110	300	1.98	112.2	301.7	50.0	110.2	28.05
110	600	1.98	111.6	601.8	49.5	109.6	54.75
110	900	1.99	112.0	903.5	50.2	110.0	81.18
110	1200	1.99	112.3	1201.9	50.4	110.3	107.75
110	1500	1.99	112.2	1500.3	49.5	110.2	134.31
150	300	1.98	151.7	301.8	49.8	149.7	26.16
150	600	1.98	151.6	602.7	50.0	149.6	52.73
150	900	1.98	152.2	903.6	49.6	150.2	79.16
150	1200	1.97	151.7	1203.7	49.6	149.7	105.46
150	1500	1.99	152.0	1499.5	49.9	150.0	131.48
190	300	1.99	190.3	298.5	49.9	188.3	24.46
190	600	2.07	192.0	599.7	49.5	189.9	50.71
190	900	2.09	191.4	908.7	50.4	189.3	76.83
190	1200	1.92	192.8	1194.7	49.6	190.9	102.99
190	1500	1.95	191.6	1514.2	50.1	189.7	128.58

A.2.2 Example report

Test time: 2020-03-25 11:05
Test location: City, State or Province, Country
Unit Tested: ACME Series XYZ Fixed Displacement Motor, model 123, serial ABCD1234
Shaft speed range: [300 rev/min, 1500rev/min]
Differential pressure range: [0bar, 190bar]
Measurement Accuracy Class: A
Test circuit filtration details: 23 micron
Description of test circuit:

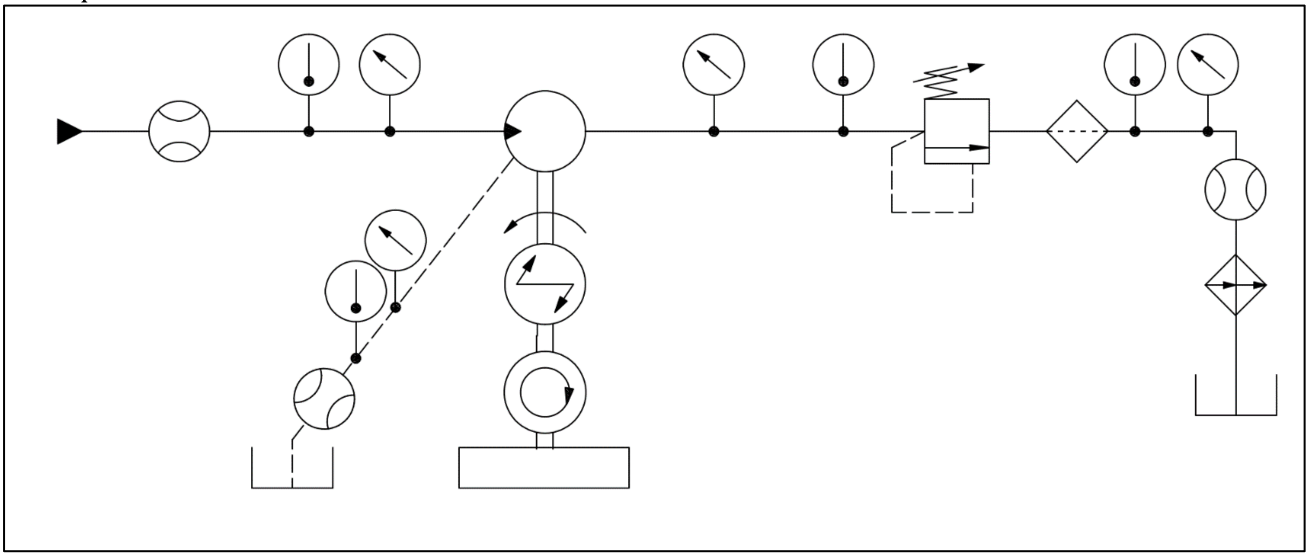
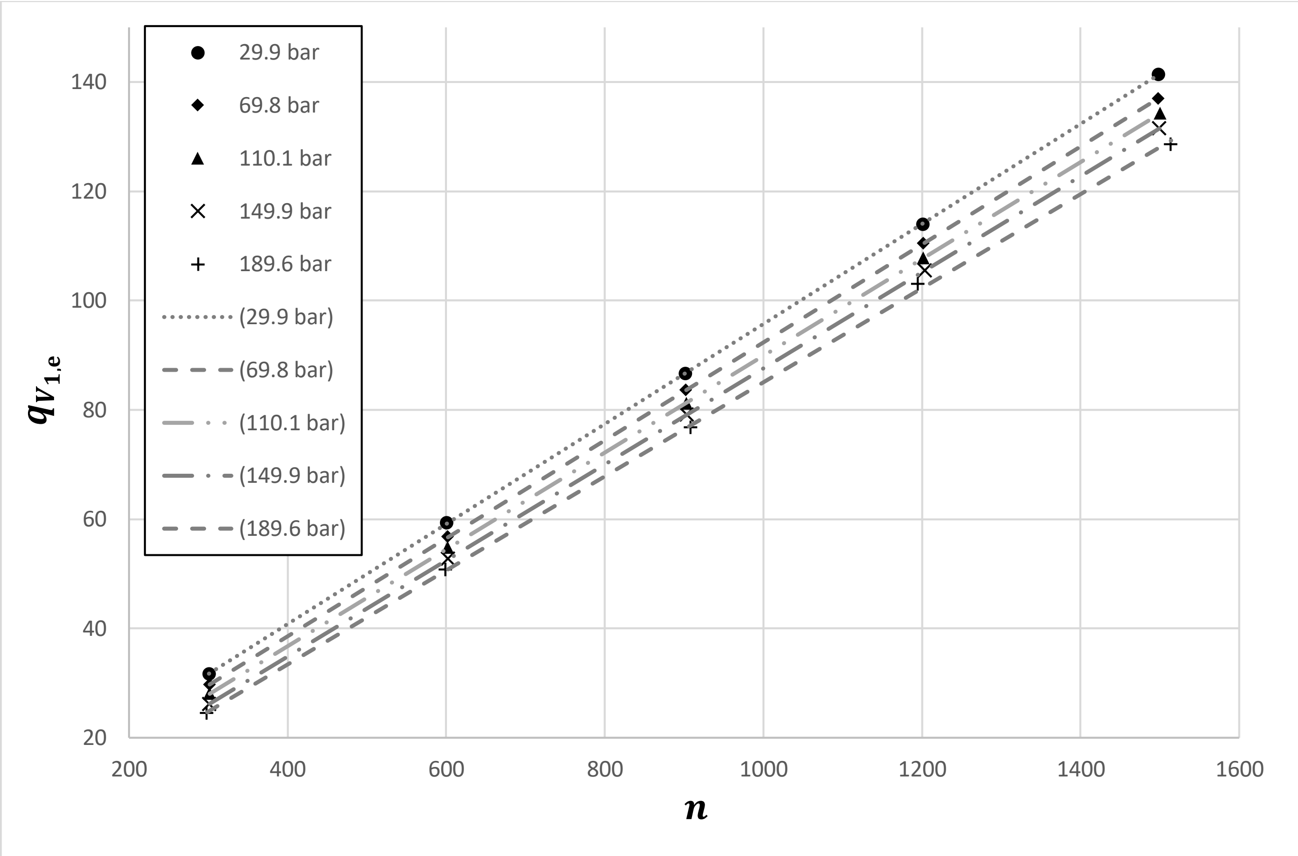


Figure A.4—Example test circuit for motor unit

Nominal ambient temperature: 20℃
Nominal inlet temperature: $\theta_1 = 50℃$
Test fluid: ABCD-OIL 46, kinematic viscosity of $30cSt$ at 50℃, density of $860kg/m^3$ at 50℃
Nominal case pressure: $p_\mathrm{d} = 1.25bar$
Time interval for flow rate measurement: $10sec$
Method for calculating derived displacement:
Both Two-step Toet Method and the One-step Toet Method were used to calculate derive displacement as shown in A.2.2.1 and A.2.2.2 respectively.

A.2.2.1 Two-step Toet Method Results

Figure A.5 and Figure A.6 show the graphs for the first and second steps respectively. Table A.5 and Table A.6 are the results of the first and second step respectively.



Key

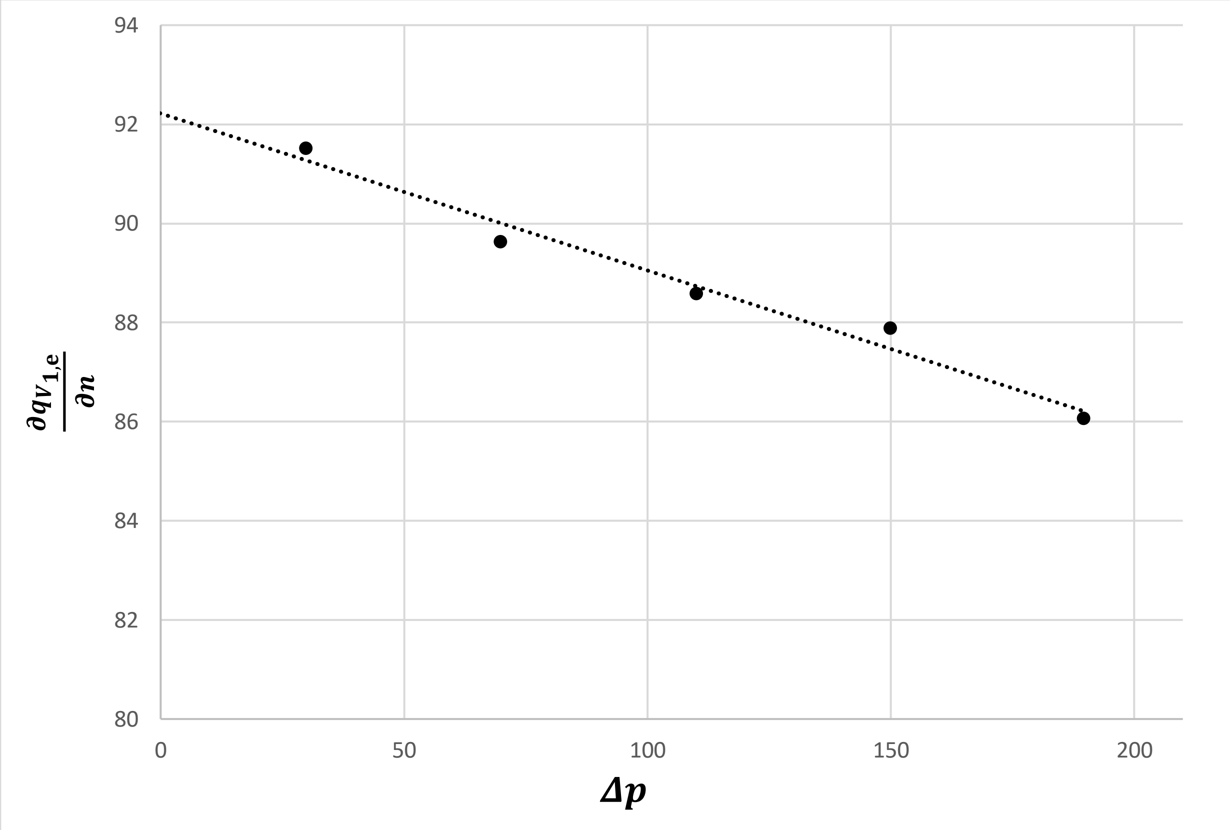
$q_{V_{1,e}}$ high pressure flow at the outlet of the pump, L/min.

n shaft speed, rev/min.

Figure A.5—First step of Two-step Toet Method for motor example

Table A.5—Results for first step for the motor example

Nominal Differential pressure [bar]	Differential pressure Average [bar]	Slope of Fitted Line [cm³/rev]	Intercept of Fitted Line [L/min]	Equation	Correlation Coefficient R^2
30	29.9	91.523	-1.4390	$q_{v_{e,1}}$=91.523*n/1000+(-1.4390)	0.99999200
70	69.8	89.638	-2.3146	$q_{v_{e,1}}$=89.638*n/1000+(-2.3146)	0.9999964
110	110.1	88.586	-2.8567	$q_{v_{e,1}}$=88.586*n/1000+(-2.8567)	0.9999923
150	149.9	87.896	-2.6920	$q_{v_{e,1}}$=87.896*n/1000+(-2.6920)	0.9999985
190	189.6	86.061	-1.0130	$q_{v_{e,1}}$=86.061*n/1000+(-1.0130)	0.9989963



Key

$\frac{\partial q_{V_{1,e}}}{\partial n}$ displacement

Δp differential pressure

Figure A.6—Second step of Two-step Toet Method for motor example

Table A.6— Results of second step for motor example

Slope [(cm³/rev)/bar]	Intercept, V_i [cm³/rev]	Equation	Correlation Coefficient R^2
-0.03171	92.222	V=-0.03170*Δp +92.222	0.974343114

A.2.2.2 One-step Toet Method Results

The results for the one step method are shown in Table A.7.

Table A.7—Results for the One-step Toet Method for the motor example

C_1 [(cm³/rev)/(rev/min*bar)]	C_2 [Lpm/bar]	C_3, V_i [cm³/rev]	C_4 [L/min]	Equation	Correlation Coefficient R^2
-0.0313	-0.03390	92.198	5.0912	$q_{v_{e,1}}$= -0.0313*n/1000*Δp +(-0.03390)*Δp +92.198*n/1000 +(5.0912)	0.999875546

Annex B
(informative)
Derivation of One-step Toet Method

B.1 Derivation of One-step Toet Method

The One-step Toet Method synthesizes the Two-step Toet Method using the underlying mathematical principles applied in one-step. The second step of the Two-Step Toet method implies a mixed derivative partial differential equation shown in B.1.

$$\frac{\partial\left(\frac{\partial q_{V_e}}{\partial n}\right)}{\partial \Delta p} = \frac{\partial^2 q_{V_e}}{\partial \Delta p \partial n} = C_1 \qquad \text{B.1}$$

Where:
C_1 is the constant slope
q_{V_e} represents $q_{V_{2,e}}$ or $q_{V_{1,e}}$

Integrate both sides with respect to Δp and add an integration constant:

$$\int \frac{\partial\left(\frac{\partial q_{V_e}}{\partial n}\right)}{\partial \Delta p}\, d\Delta p = \int C_1\, d\Delta p + C_3 \qquad \text{B.2}$$

$$\frac{\partial q_{V_e}}{\partial n} = C_1 \Delta p + C_3$$

Where:
C_3 is an integration constant

Integrate with respect to n, include a $C_2 \Delta p$ term (similar to an integration constant) because differentiating by n would cancel it, and add an integration constant:

$$\int \frac{\partial q_{V_e}}{\partial n}\, dn = \int (C_1 \Delta p + C_3) dn + C_2 \Delta p + C_4 \qquad \text{B.3}$$

$$q_{V_e} = C_1 \Delta p \cdot n + C_2 \Delta p + C_3 n + C_4$$

Where:
C_2 is arbitrary constant
C_4 is an integration constant

Finally, a boundary condition is applied to keep the equation as physically relevant as possible. When the shaft speed and differential pressure are 0, the high pressure flow is zero, $q_{V_\mathrm{e}}(n = 0, \Delta p = 0) = 0$ (Formula B.4):

$$q_{V_\mathrm{e}} = C_1 \cdot n \cdot \Delta p + C_2 \cdot \Delta p + C_3 \cdot n \qquad \text{B.4}$$

The interpretation of each constant is

- C_1 and C_2 represent loss coefficients
- C_3 represents the derived displacement, V_i. $V_i = C_3$
- C_4 is a bias term. It is not physical. At Δp=0 and $n = 0$ and if $C_4 \neq 0$, then the pump or motor would still have high pressure flow.

In fitting a linear equation, keeping the bias term, C_4, is best practice. Therefore, the equation to use to describe the One-Step Toet Method is

$$q_{V_e} = \underbrace{C_1 \cdot n \cdot \Delta p + C_2 \cdot \Delta p} + \underbrace{C_3 \cdot n} + \underbrace{C_4} \qquad \text{B.5}$$

Where:

$C_1 \cdot n \ \cdot \Delta p + C_2 \cdot \Delta p$ are loss terms $C_3 \cdot n$ is the theoretical term, $V_i n$ C_4 is the bias term	

Annex C
(informative)
Residual and regression metrics

C.1 Residual and regression metrics

Evaluation of the residuals from regression analysis is helpful for creating useful models and assessing the validity of derived displacement:

a) The root mean square error (RMSE)—also referred to as standard error—is a measure of the standard deviation of the residuals of the regression. The smaller the value, the better the model fits the data. The RMSE is a metric regularly available as an output of a spreadsheet program or statistical software. The equation for RMSE is shown in Formula C.1 to clarify its definition.

$$S = \sqrt{\frac{\sum_{i=1}^{m}(y_i - \hat{y}_i)^2}{m - r - 1}} \quad \text{(C.1)}$$

where

S is Root Mean Square Error (RMSE)

m is the number of samples

r is the number of fitted linear terms not including the constant. For the One-Step Toet Method, r is three.

y_i is ith measured value

$\hat{y}_i$ is ith fitted response

b) The correlation coefficient (R^2) is a metric for assessing how well a model fits the data. It measures the proportion of variation in the data for which the model accounts. An R^2 of 0.95 is expected for derived displacement with values approaching or slightly exceeding 0.99 observed in some cases.

Regular or cyclic patterns in the residuals indicates that the residuals contain unmodeled systematic errors. Analyze the residuals for patterns to identify problems with the test and regression.

Annex D
(informative)
Formulas for fitting a line

D.1

The regression equation for a line is shown in Formula D.1. The intercept, a, and the slope, b, are found via Formula D.2 and Formula D.3 respectively. Formula D.2 and Formula D.3 are based on ordinary least squares regression for fitting a line.

$$y = a + bx \tag{D.1}$$

where
x is the independent variable
y is the dependent variable
a is the intercept
b is the slope

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{m(\sum x^2) - (\sum x)^2} \tag{D.2}$$

where
x is the measured independent variable
y is the measured dependent variable
a is the intercept
m is the number of points

$$b = \frac{m(\sum xy) - (\sum x)(\sum y)}{m(\sum x^2) - (\sum x)^2} \tag{D.3}$$

where
x is the measured independent variable
y is the measured dependent variable
b is the slope
m is the number of points

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