A Fast Lumped Parameter Approach for the Prediction of Cavitation in Gerotor Pumps

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Outline

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Introduction

• Aeration and Vapor cavitation affects the operation of many fluid power systems.

• Negative effects of Cavitation:
  – Reduced flow capacity
  – Noise generation
  – Erosion in Components
  – Increased Vibrations

Cavitation damage in a pump
(Brennen C.E., Cavitation and Bubble Dynamics, 2013)
State of the art

Modeling approaches for Cavitation

• CFD modeling approaches
  – Singhal et. al, 2002
  – Zwart et. al, 2004
  – Schnerr & Sauer, 2001

• Lumped Parameter Approach
  – Vacca et. al, 2013
  – Ivantysynova et al., 2012
  – SA Imagine, 2007

• Semi-empirical approaches
  • Not suitable for complex systems modeled using Lumped Parameter Approach
State of the art

**--Wylie, 1978**

\[
E = \frac{E_0}{1 + \alpha \left( \frac{p_0}{p} \right)^{1/\lambda} \left( \frac{E_0}{\lambda p} - 1 \right)}
\]

**--Ruan, 2006**

\[
E = \begin{cases} 
\frac{E_0}{\alpha p_0 E_0 \left( \frac{1}{p^2} - \frac{1}{p_c^2} \right) + 1} & (p_0 < p < p_c) \\
E_0 & (p \geq p_c)
\end{cases}
\]

**--Nykanen, 2000**

\[
E = \alpha \left( \frac{p_0}{p} \right)^{1/\lambda} + \frac{1 - \alpha}{1 + \frac{p - p_0}{E_0}}
\]

All of these models do not take vapor cavitation into account and assume a constant air content.
State of the art

- Models accounting for variable air content:
  - Vacca, 2009
  - LMS Imagine SA, 2009
  - Gholizadeh, 2012
Modeling of Gerotor pumps

Lumped Parameter Approach:

- The system is divided in several control volumes.
- The physical quantities pertaining to fluid in a control volume are assumed to be uniform.
- Pressure is evaluated using Conservation of Mass.
- Flow is computed from the orifice equation.
- The system of non-linear Differential Equations is solved using software AMESim.
Modeling of Gerotor pumps

- The model consists of several modules – geometric, fluid dynamic, forces, journal bearing module, rotors radial movement.

- The geometric module programmed in C++ computes the geometrical features for different angles of rotation of TSVs.

- These are used by other modules, to evaluate the pressures, flow rates and forces on different elements.

Pellegrin et al., 2016
Lumped Parameter Approach for Cavitation

• The multiphase flow is being modeled as flow of a homogenous multicomponent fluid.

• The density ($\rho_i$) of the resulting fluid depends on the void fraction ($\alpha_i$) of the individual phases.

• The existing lumped parameter approach needs to be modified to account for the compressibility effects of the multicomponent fluid.

\[
\alpha_i = \frac{V_i}{V}, \quad i = g, v, l
\]

\[
\rho = \sum \alpha_i \rho_i, \quad \sum \alpha_i = 1
\]
Governing Equations

- Pressure built up equation is expressed in terms of the Effective Bulk Modulus ($E$).

- Vapor and Gas phases are assumed to undergo polytropic processes.

- The bulk moduli of gas and vapor phases depend on pressure.

- The effective Bulk Modulus of the homogenous fluid depends on the individual void fractions as well as pressure.

$$E = \left( \sum \frac{\alpha_i}{E_i} \right)^{-1}$$

$$E_i(t) = \lambda_i p(t), \quad i = g, v$$
Governing Equations

- Flow rate (Q) is derived from the compressible Bernoulli equation.
- The effective flow rate depends on Mass fractions ($f_i$) and pressures across the orifice.

$$v = \sqrt{\int_{P_u}^{P_d} \frac{dp}{\rho}}$$

$$\frac{1}{\rho} = \sum f_i \frac{\rho_i}{\rho_i}, \quad f_i = \frac{\alpha_i \rho_i}{\rho}$$

$$Q = C_q A \mid v \mid \text{sign}(p_u - p_d)$$
Closure Relations

1. Under simplified assumptions, the Rayleigh Plesset equation was solved by Singhal et. al.

2. The gas content at equilibrium is given by Henry’s law.

3. The closure relations were derived in Zhou et. al from the full Cavitation model derived by Singhal et. al.

\[ R \frac{d^2 R}{dt^2} + \frac{3}{2} \left( \frac{dR}{dt} \right)^2 + \frac{4\nu}{R} \frac{dR}{dt} + \frac{2S}{\rho_l R} = \frac{p_B(t) - p_\infty(t)}{\rho_l} \]

\[
\frac{df_v}{dt} = \begin{cases} 
  \frac{k_{11}}{\tau} (1 - f_{g0} - f_v) \sqrt{p - p_v} & (p \leq p_v) \\
 -\frac{k_{12}}{\tau} f_v \sqrt{p_v - p} & (p > p_v)
\end{cases}
\]

\[
\frac{df_g}{dt} = \begin{cases} 
  \frac{k_{21}}{\tau} (f_H - f_g) \sqrt{p_0 - p} & (f_g \leq f_H) \\
 -\frac{k_{22}}{\tau} f_g \sqrt{p - p_0} & (f_g > f_H)
\end{cases}
\]

\[
f_H = \begin{cases} 
  f_{g0} & (p \leq p_v) \\
  f_{g0} \left(1 - \frac{p - p_v}{p_0 - p_v}\right) & (p_v < p \leq p_0) \\
  0 & (p > p_0)
\end{cases}
\]
Phase Transport

• In modeling the phase transport, it is assumed that the phases are advected to a chamber located downstream from a chamber located upstream.

\[
\frac{df_j}{dt} = \left( \frac{df_j}{dt} \right)_p + \frac{1}{\rho} \left( \sum \rho_i Q_j f_{j,i} \right), \quad j = v, g
\]

Phase Change  Advection

• The additional term arising from volume change reported by Zhou et. al was omitted from phase transport.
Prediction of Phases

- The phase transport across connections from multiple chambers to one is obtained from mass conservation:

\[ fQ_\rho = \sum_{Q_i>0} f_i Q_i \rho_i + \sum_{Q_i<0} f Q_i \rho \]

\[ f = \frac{\sum c_i Q_i \rho_i}{\sum c_i Q_i \rho} \]
Experimental Setup

• The reference pump considered for validation of model is a Magna 9/10 pump (5cc) used for Engine lubrication.

• A calibrated orifice was installed at the end of pump outlet and inlet.

• The experimental tests were performed at Hydraulic Laboratory of the University of Naples Federico II.

• The test was performed for inlet orifice diameters of 5mm, 7mm and 15mm operating at speeds of 1000 rpm, 2000 rpm and 4000 rpm.

Pellegri et. al
Experimental Setup

Reference Pump (Pellegrini et al.)

Legend:
- T = Torque Meter
- P₁ = Mean Delivery Pressure Transducer
- P₂ = Mean Suction Pressure Transducer
- P₃ = Delivery Chamber Pressure Transducer
- P₄ = Suction Chamber Pressure Transducer
- Q = Flow Meter
- R₁ = Suction Lamination Valve
- R₂ = Suction Calibrated Orifice
- R₃ = Delivery Calibrated Orifice
- R₄ = Delivery Lamination Valve
## Results

**Chosen Empirical Parameters:**

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<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$k_{11}$</td>
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<tr>
<td>$k_{12}$</td>
<td>0.7</td>
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<tr>
<td>$k_{21}$</td>
<td>0.2</td>
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<tr>
<td>$k_{22}$</td>
<td>0.7</td>
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<tr>
<td>$\tau$</td>
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</tr>
</tbody>
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Instantaneous void fraction of the gas inside TSV
Conclusion

• A lumped parameter approach for the simulation of a Gerotor pump was presented.

• The approach is capable of considering the instantaneous variation in gas and vapor phases.

• The model accounts for the effects of compressibility in flows through hydraulic connections.